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Effect The Change in Bosons on The Properties of Xenon Isotopes

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Abstract: There are two kind of bosons: holes and particles depending on the nearest closed shell or middle closed shell for neutrons or protons. In this article, the kind of bosons for even-even Xe isotopes have been changed from holes to particles for mass number 132 to 140. Energy levels and electric transitions probability calculated in IBM-1 and IBM-2 models, and potential energy surface to determine the limit of these isotopes and the deformation in them. There are a good matching between two models and experimental data.

Keywords: interacting boson model– 1 and 2, energy levels, electric transitions probability, potential energy surface.

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1 Introduction

A nuclear model in 1974 was proposed by Arima and Iachello, called (IBM) interacting boson model of nuclear structure. The IBM has been applied extensively to correlate the collective properties of even-odd nuclei by coupling a single-particle (fermion) to the even-even core and even-even nuclei (Iachello and Arima, 1978; Iachello, 1980; Sharrad *et al.*, 2012). The models IBM-1 and IBM-2 are restricted to nuclei with even numbers of protons and neutrons. In order to fix the number of bosons one takes into account that both types of nucleons constitute closed shells with particle numbers ..28, 50, 82 and 126 (in the analogy with the Mendeleev periodic table one would expect that there are the discontinuities in the dependence of various measurable quantities on N or Z when oscillator shells are filled. However, these discontinuities were experimentally observed not for these numbers but for so called magic numbers N(Z) = 2, 8, 20, 28, 50, 82 and 126)(Kvasil, 2013).

Provided that the protons fill less than half of the furthest shell the number of the corresponding active protons has to be divided by two in order to obtain the boson number N_p attributed to protons. If more than half of the shell is occupied the boson number reads N_p = (number of holes for protons)/2. By treating the neutrons in an analogous way, one obtains their number of bosons N_n . In the IBM-1 the boson number N is calculated by adding the partial numbers i.e. $N = N_p + N_n$(Pfeifer, 1998).

2 interacting boson models

Interacting boson model with simple form, known as IBM-1, describes the low-lying collective excitations of an even-even nucleus in terms of the s(L=0) and d(L=2) bosons. Casten and Warner have given a comprehensive review of this model and its application to the transition region. The Hamiltonian of this model contains two terms of one body interactions (E_s and E_d where they are the single boson energies) and seven terms of two-body interactions C_L (L = 0,2,4), v_L (L = 0,2), u_L (L = 0,2) where they are the two boson interactions. For a fixed boson number N, only one of the one-body term and five of the two body terms are independent, as it can be seen by noting $N = n_s + n_d$(Al-Khudair, 2018) The second version of the interacting boson model IBM-2, was introduced in 1978 as a modification of the first model (IBM-1), through introducing the concept of degrees of freedom. This made it possible to distinguish between wave function for protons and wave function for neutrons. Thus, it becomes possible to obtain some of the energy levels that could not be determined by IBM-1, these levels are called mixed symmetry states (MSS). The mixing case between wave function for protons and wave function for neutrons can affect the site of energy level for the rest of the energy levels(Al-sadi *et al.*, 2015).

3 The Hamiltonians

3.1 IBM-1

The IBM-1 Hamiltonian can be grouped into different boson-boson interactions as(Warner, 1988):

$$\hat{H} = E\hat{n}_d + a_0\hat{p}.\hat{p} + a_1\hat{L}.\hat{L} + a_2\hat{Q}.\hat{Q} + a_3\hat{T}_3.\hat{T}_3 + a_4\hat{T}_4.\hat{T}_4 \quad (1)$$

$E=E_d - E_s$ is the boson energy

The parameters a_0, a_1, a_2, a_3 and a_4 are the strength of the pairing, angular momentum, quadrupole, octoupole and hexadecapole interaction between the bosons, respectively.

$\hat{n}_d=(d^{\dagger}.\tilde{d})$: the total number of d-boson operator and $\hat{p}=1/2\left[(\tilde{d}.\tilde{d})-(\tilde{s}.\tilde{s})\right]$: the pairing operator.

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$\hat{L}=\sqrt{10}\,[d^\dagger \times \tilde{d}]^{(1)}$: the angular momentum operator, $\hat{Q}=[d^\dagger \times \tilde{s}+s^\dagger \times \tilde{d}]^{(2)}+\chi[d^\dagger \times \tilde{d}]^{(2)}$: the quadrupole operator and χ: the parameter of quadrupole structure (between 0 and $\pm\frac{\sqrt{7}}{2}$).

$\hat{T}_m = [d^\dagger \times \tilde{d}]^{(m)}$: the octoupole ($m$=3) and hexadecapole ($m$=4) operator.

3.2 IBM-2

Interacting boson model-2 introduce all parameters that are effective in the Hamiltonian operator(Abood and Najim, 2013):

$$\hat{H} = E_d(\hat{n}_{d\nu} + \hat{n}_{d\pi}) + \kappa(\hat{Q}_\nu.\hat{Q}_\pi) + V_{\nu\nu} + V_{\pi\pi} + M_{\nu\pi} \quad (2)$$

E_d: the energy difference between s and d boson.

$n_{d\rho}$ represents the number of d bosons, where ρ goes along with ν(neutron) and π(proton) bosons.

The second term refers to the quadrupole–quadrupole interaction between neutron and proton with strength k, where the quadruple operator Q_ρ can be written as(Al-sadi *et al.*, 2015):

$$Q_\rho = [\, d_\rho^\dagger s_\rho + s_\rho^\dagger d_\rho]^{(2)} + x_\rho[d_\rho^\dagger\, d_\rho]^{(2)} \quad (3)$$

χ_ρ: the quadrupole deformation parameter for neutron and proton.

The $V_{\pi\pi}$ and $V_{\nu\nu}$ which refer to the interaction between like boson, using to improve the fit with experimental energy spectra and they are given by:

$$V_{\rho\rho} = \frac{1}{2}\sum_{L=0,2,4} C_L^\rho([d_\rho^\dagger d_\rho^\dagger]^{(L)}\ .[\tilde{d}_\rho \tilde{d}_\rho]) \quad (4)$$

The Majorana term $M_{\nu\pi}$ has the parameters of ξ_1 , ξ_2 and ξ_3 which can be expressed as:

$$M_{\nu\pi} = \frac{1}{2}\xi_2([\, s_\nu^\dagger d_\pi^\dagger - d_\nu^\dagger s_\pi^\dagger]^{(2)}.[s_\nu d_\pi - d_\nu s_\pi\,]^{(2)}) - \sum_{k=1,3}\xi_k([d_\nu^\dagger d_\pi^\dagger\,]^{(k)}.[d_\nu d_\pi]^{(k)}) \quad (5)$$

4 Electric transitions

Most *B(E2)* values known to date were measured by coulomb excitation. In order to do so, one has to specify the transition operators in terms of the boson operators. In lowest order, it is assumed that the transition operators will contain only one-body terms. Clearly, the most general form of such an operator in IBM-1 can be given by(Iachello and Haven, 1987):

$$T_m^l = \alpha_2\delta_{l2}[d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d}]_m^2 + \beta_l[d^\dagger \times \tilde{d}]_m^l \quad (6)$$

It is clear that the first term can be presented only in the case of l =2. In the special cases of electric monopole, quadrupole and hexadecapole transitions, the specific form of the transition operator is respectively, γ_0, α_2 and β_l (l = 0, 1, 2, 3, 4) which are parameters specifying the various terms in the corresponding operators. Then the electric quadrupole transition is(Motelb, 2012; Hussain *et al.*, 2017):

$$T_m^{E2} = \alpha_2[d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d}]_m^2 + \beta_2[d^\dagger \times \tilde{d}]_m^2 = \alpha_2([d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d}]_m^2 + \chi[d^\dagger \times \tilde{d}]_m^2) = e_B\hat{Q} \quad (7)$$

where $\alpha_2 = e_B$ (effective charge) and ($\beta_2 = \chi\alpha_2$).

The electric quadrupole reduced transition probabilities B (E2) is defined as:

$$B(E2), L_i \rightarrow L_f = \frac{\langle L_f | T(E2) | L_i \rangle^2}{2L_i + 1} \quad (8)$$

where L_i and L_f are angular momentum of the initial and final states respectively.
The matrix element of electric transition T in IBM-2 is:

$$\hat{T}^{(E2)} = \hat{T}_\pi^{(E2)} + \hat{T}_\nu^{(E2)} \qquad \text{where} \qquad \hat{T}_\pi^{(E2)} = e_\pi Q_\pi\,,\ \hat{T}_\nu^{(E2)} = e_\nu Q_\nu \quad (9)$$

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e_π , e_ν: the effective charge for each of the proton and the neutron, the unity (eb) is dependent on the number of bosons protons and neutrons(N_π, N_ν). After calculating the T^{E2} transition operator for proton and neutron from equation (8), the reduced transition probability *B(E2)* can be calculated by using the same relations in equation (9). Generally, in IBM-2 the *B(E2)* for the transition($2_1^+ \rightarrow 0_1^+$)is associated with $\boldsymbol{e_\rho}$ and N_ρ through the following relations(Warner, 1988):

$$B(E2, 2_1^+ \rightarrow 0_1^+) = \frac{(N_\pi e_\pi + N_\nu e_\nu)^2}{M}$$

where M is the parameter that depends on the number of bosons N_ρ, and it takes different values depending on the three types of the dynamical symmetries U(5), SU(3) and O(6), which are given below(Iachello and Arima, 1978):

$$M = \begin{cases} U(5): & \dfrac{1}{N_\rho} \\ SU(3): & \dfrac{(2N_\rho + 3)}{5N_\rho} \\ O(6): & \dfrac{(N_\rho + 4)}{5N_\rho} \end{cases} \tag{10}$$

5 Potential energy surface

The potential energy surface $PES\ (N, \beta, \gamma)$ gives a final shape to the nucleus that corresponds to the function of Hamiltonian, as the equation(Kvasil, 2013):

$$PES(N, \beta, \gamma) = \langle N, \beta, \gamma | H | N, \beta, \gamma \rangle / \langle N, \beta, \gamma | N, \beta, \gamma \rangle \tag{11}$$

The energy surface, as a function of $\beta$and γ, has been given by:

$$PES\,(N, \beta, \gamma) = \frac{NE_d}{(1+\beta^2)} + \frac{N\,(N+1)}{(1+\beta^2)^2}(A_1\beta^4 + A_2\beta^3\cos(3\gamma) + A_3\beta^2 + A_4 \tag{12}$$

where the A_i's are values, we get them from the equation from results of energy levels.

β: a measure of the total deformation of nucleus, when $\beta = 0$ the shape is spherical, and be distorted when $\beta \neq 0$, and γ is the amount of deviation from the symmetry and correlates with the nucleus, if γ=0 the shape is prolate, and if γ = 60 the shape becomes oblate(Abood *et al.*, 2013; Al-Khudair *et al.*, 2014).

The following equations represented potential energy surface for three dynamical symmetries(A. Subber, W. Hamilton, 1987):

$$E\,(N, \beta, \gamma) \propto \begin{cases} U(5): & E_d N \dfrac{\beta^2}{1+\beta^2} \\ SU(3): & kN\,(N-1)\dfrac{\frac{3}{4}\beta^4 - \sqrt{2}\beta^3\cos 3\gamma + 1}{\left(1+\beta^2\right)^2} \\ O(6): & k'N\,(N-1)\left(\dfrac{1-\beta^2}{1+\beta^2}\right)^2 \end{cases} \tag{13}$$

where $K \propto a_2$ and $\acute{K} \propto a_0$ in equation (1).

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6 The results

The energy levels can be calculated after determination the limit of the isotopes in the research ($^{132\text{-}140}$Xe), therefore we calculate the ratio between the excited levels especially the ratio of (4_1^+) energy level and (2_1^+) energy level which approximately equal to(2.1, 2, 1.8 and 2.2) respectively and the ratio of (6_1^+) and (2_1^+) which they are between (3.2, 2.5,2.6 and 3.7) for these isotopes, this mean these isotopes in limit U(5) for $^{132\text{-}138}$Xe and transition between U(5) and O(6) limit for ^{140}Xe and its Hamiltonian's parameters have been shown in table 1 and 2 for two models.

The effective charge is E2SD=α_2 and E2DD=β_2, where $\beta_2 = (-0.7/5)\alpha_2$, $-\sqrt{7/2}\,\alpha_2$ and equal zero in SU(5), SU(3) and O(6) respectively. The converter coefficient between (e^2b^2) and (W.u) is:

$$B(E2)w.u = \frac{B(E2)e^2b^2}{5.943\times10^{-6}A^{4/3}e^2b^2}$$

Table 1, represents the parameters of Hamiltonians in equation(1), with E2SD=(0.1515, 0.144, 0.156 and 0.127) and E2DD=(-0.106, -0.1, -0.11 and 0) for $^{132\text{-}140}$Xe respectively. Table 2, represents the parameters of Hamiltonian in equations(2, 3, 4 and 5) with $\boldsymbol{x_\pi}$=0.8 and $\boldsymbol{C_L^\pi}$=0,0,0 for all isotopes.

Table 1. The parameters of Hamiltonian in IBM-1(MeV unit)

The Isotopes	Bosons Number	E	a_0	a_1	a_2	a_3	a_4
^{132}Xe	4	0.28324	0	0.011746	0	-0.2	0.33
^{134}Xe	3	0.8793	0	0.0047	0	-0.0213	-0.017
^{138}Xe	3	0.3026	0	-0.0103	0	-0.337	0.22
^{140}Xe	4	0.435	0.314354	0.00363	0	0.2526	0

Table 2. The parameters of Hamiltonian in IBM-2(MeV unit)

The Isotopes	Neutrons Number	Protons Number	E_d	k	x_ν	Fs	Fk	C_L^ν
^{132}Xe	2	2	1.04	-0.5	-0.86	0.15	-0.06	-0.5,-1.6, -1.6
^{134}Xe	1	2	0.95	-0.1	-0.73	0.18	-0.06	0,0,0
^{138}Xe	1	2	0.69	-0.1	0.1	0.24	-0.06	0,0,0
^{140}Xe	2	2	0.48	-0.06	0.5	0.46	-0.06	5,0.4,0.1

We note from the parameters in tables 1 and 2, there are change in them because there are change in boson number from holes boson for ^{132,134}Xe to particles boson for ^{138,140}Xe, but constant proton bosons as particles boson for all isotopes. Figure 1 shows the energy levels for the isotopes, compression between the models with experimental data(exp)(*Nuclear Data, update 2019*; Pfeifer, 1998):

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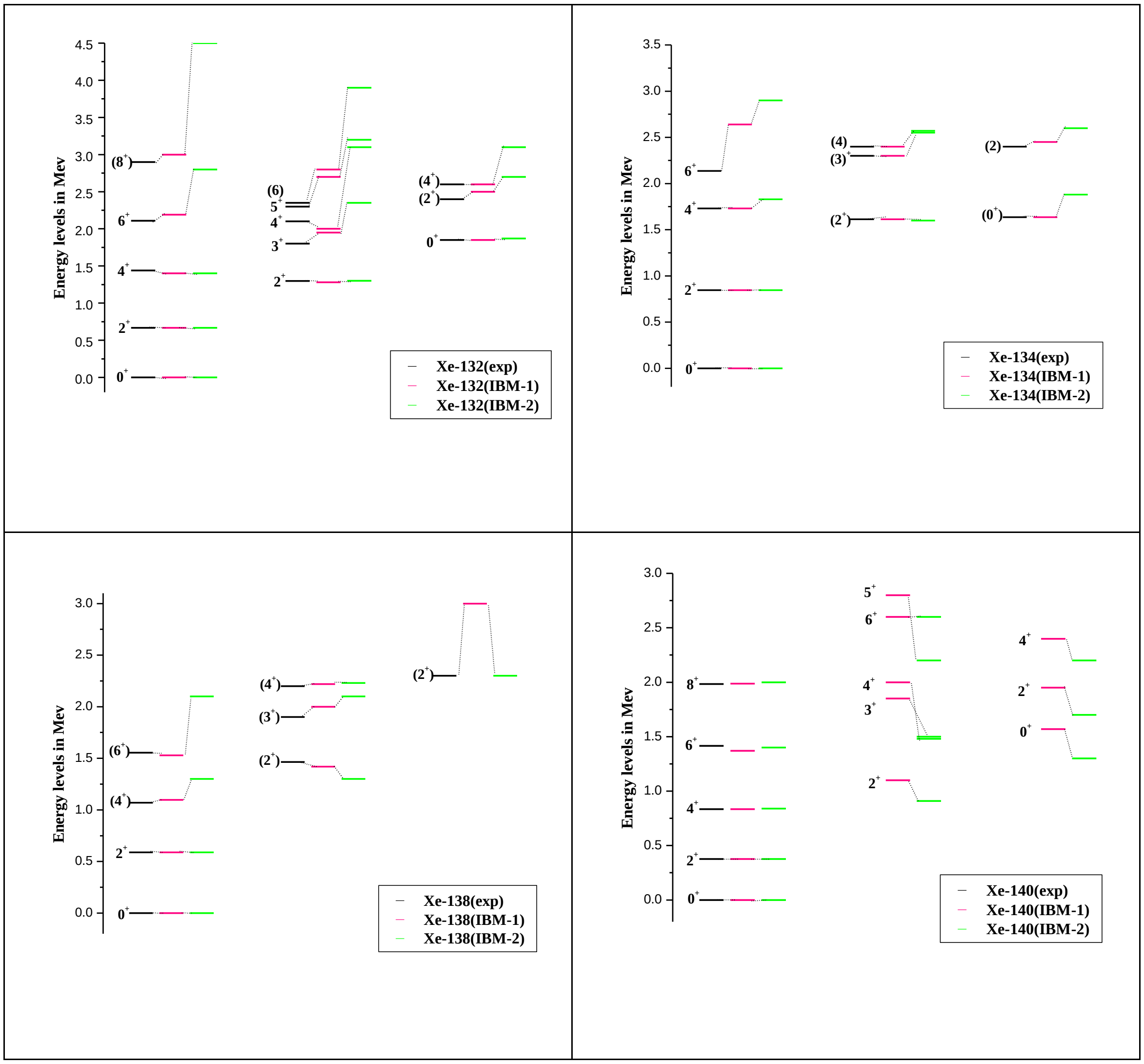

Figure 1. The energy levels for $^{132-140}$Xe compression the models with experimental data

There are many uncertain levels in $^{132-138}$Xe, and a good agreement between models and experimental data for low levels in ground band (0^+, 2^+, 4^+,….) and the other bands, and ^{140}Xe has experimental ground band only but we gotten in the two models for another two bands.

Tables 3 and 4 represent the electric transition probability for ^{132,134}Xe and ^{138,140}Xe respectively for some transitions and compared between experimental and the two models with rapprochement. There are

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decrease in transitions with decrease in bosons number for ^{134}Xe and ^{138}Xe, as transitions $6_2 \rightarrow 6_1$ and $8_1 \rightarrow 6_1$, because the two models depend on the bosons number.

Table 3. The electric transitions (e^2b^2 unit) with positive parity for ^{132,134}Xe

The Isotopes		**^{132}Xe**			**^{134}Xe**	
$I_i \rightarrow I_f$	**exp**	**IBM-1**	**IBM-2**	**exp**	**IBM-1**	**IBM-2**
$2_1 \rightarrow 0_1$	0.09	0.08	0.1	0.06	0.054	0.07
$2_1 \rightarrow 0_2$	-	0	0.0009	-	0.0145	0.012
$2_2 \rightarrow 0_1$	0.0003	0.0008	0.008	-	0	0.003
$2_2 \rightarrow 0_2$	-	0.024	0.03	-	0.007	0.0015
$2_2 \rightarrow 2_1$	0.164	0.12	0.134	-	0.07	0.05
$4_1 \rightarrow 2_1$	0.11	0.12	0.097	0.047	0.072	0.09
$4_2 \rightarrow 2_2$	-	0.063	0.054	-	0.029	0.04
$4_2 \rightarrow 4_1$	-	-	0.049	-	-	0.018
$6_1 \rightarrow 4_1$	-	-	0.095	-	-	0.06
$6_2 \rightarrow 6_1$	-	-	0.018	-	-	-
$8_1 \rightarrow 6_1$	-	-	0.068	-	-	-

Table 4. The electric transitions (e^2b^2 unit) with positive parity for ^{138,140}Xe

The Isotopes		**^{138}Xe**			**^{140}Xe**	
$I_i \rightarrow I_f$	**exp**	**IBM-1**	**IBM-2**	**exp**	**IBM-1**	**IBM-2**
$2_1 \rightarrow 0_1$	0.076	0.077	0.075	0.1	0.09	0.09
$2_1 \rightarrow 0_2$	-	0.011	0.012	-	0	0.0016
$2_2 \rightarrow 0_1$	-	0.0002	0.0003	-	0	0.0002
$2_2 \rightarrow 0_2$	-	0.008	0.01	-	0	0.0012
$2_2 \rightarrow 2_1$	-	0.095	0.086	-	0.109	0.104
$4_1 \rightarrow 2_1$	-	0.095	0.096	0.17	0.109	0.12
$4_2 \rightarrow 2_2$	-	0.035	0.004	-	0.05	0.003
$4_2 \rightarrow 4_1$	-	-	0.027	-	-	0.045
$6_1 \rightarrow 4_1$	-	-	0.06	0.095	-	0.12
$6_2 \rightarrow 6_1$	-	-	-	-	-	0.019
$8_1 \rightarrow 6_1$	-	-	-	-	-	0.076

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Finally, the parameters of potential energy surface as in table 5, these parameters are been getting from the equation 12 exists in energy levels results, and these parameters put in the program of the potential. The results are the relation between the potential, deformation and the angle of deviation.

Table 5. The parameters of potential energy surface(MeV unit)

The Isotopes	E_d	A_1	A_2	A_3	A_4
^{132}Xe	0.668	0.17	0	0	0
^{134}Xe	0.847	-0.009	0	0	0
^{138}Xe	0.589	0.108	0	0	0
^{140}Xe	0.379	0.079	0	-0.157	0

Figure 2 shows the symmetric relation between the deformation and potential energy surface and contour lines for the relation between them and the deviation angle($0 \geq \gamma \geq 60$). There are a small deformation in these isotopes because they have a few bosons number and closure from closed shell (82) for neutrons and near to U(5) limit. There are change in the distribution of potential energy from first isotope (maximum value about 3 Mev for deformation greater than 2). Second and third isotopes there are reduction in potential with deformation(maximum value 1.9 and 1.7 Mev for deformation about 2). The last isotope there is some deformation in contour lines near from 0.5 because it is translate between U(5) to O(6) limit.

6 Conclusions

The limit of these isotopes are U(5) and transition between U(5) and O(6). There is decreasing in electric transitions with decreasing in bosons number for isotopes because the approaching from stability.

There are many levels have been confirmed for $^{132\text{-}138}Xe$ and there are new levels for ^{140}Xe. Energy levels decreases with decrease of bosons number, because near the neutrons or(and) protons number from closed shell. Potential energy surface shows the translation the isotopes from symmetric vibration U(5) for $^{132\text{-}138}Xe$ to unstable rotation O(6) for ^{140}Xe.

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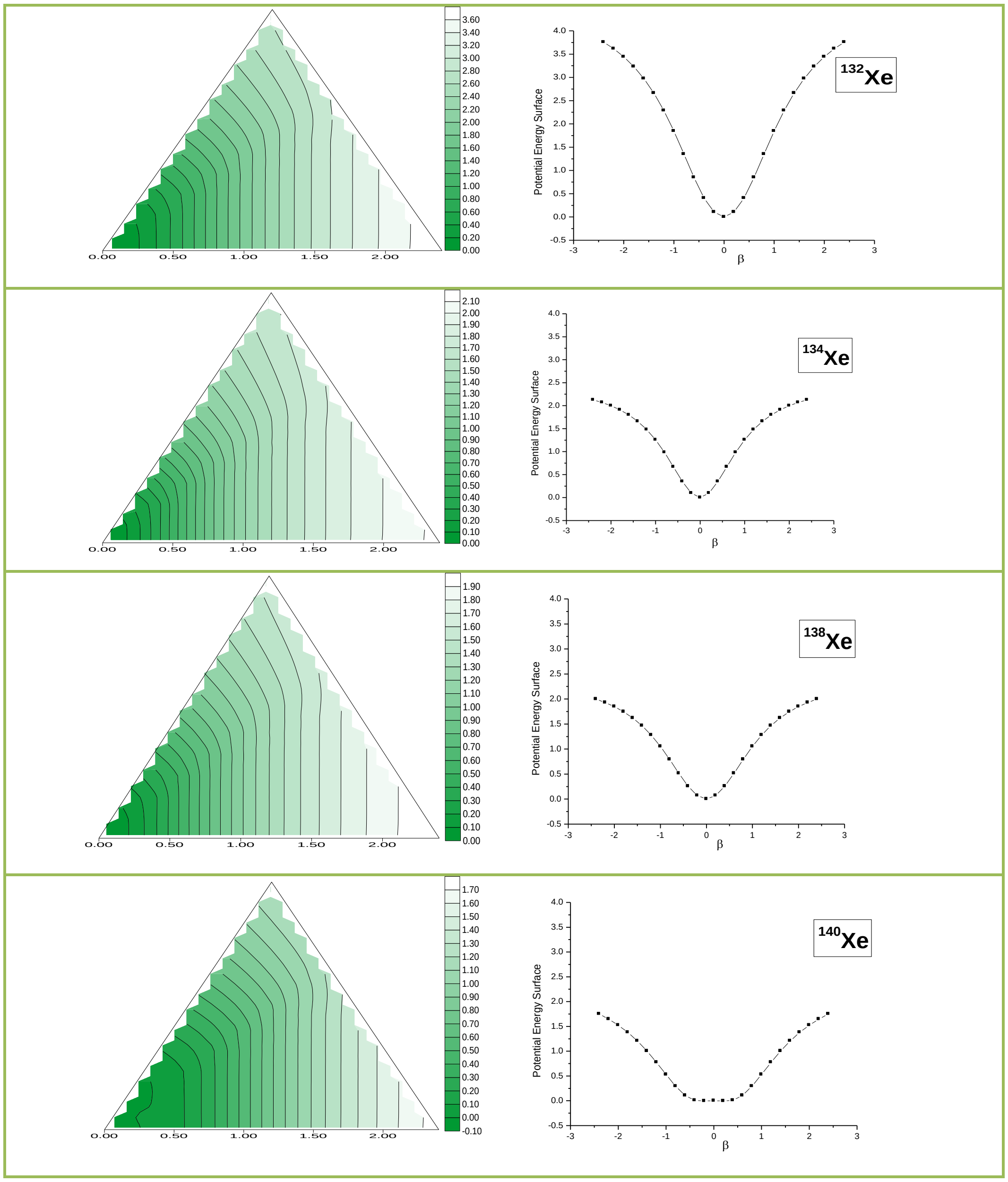

Figure 2. The symmetric relation and contour for $^{132-140}$Xe.

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